# Advances and Trends in Computational Structural Mechanics

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## Introduction

ADVANCES in computer technology have had a profound effect on structural mechanics as well as on various engineering, applied science, and mechanics disciplines. In the last two decades, a new discipline now labeled computational structural mechanics (CSM), has emerged as a blend between structural mechanics and other disciplines, such as computer science, numerical analysis, and approximation theory. Development of the modern finite-element method, which is currently the backbone of many engineering analysis systems, marks the beginning of CSM.

The application of CSM to contemporary problems typically involves a sequence of steps[53]; namely:

1) Observation of the response phenomena of interest.

2) Development of a computational model for the numerical simulation of these phenomena. This in turn includes:

a) Identification of a mathematical model that describes the phenomena, and testing the range of validity of this model;

b) Development of a discrete model, computational strategy, and numerical algorithms to approximate the mathematical model; and

c) Development and assembly of software and/or hardware to implement the computational strategy and the numerical algorithms. Successful computational models for structures are generally those based on a thorough familiarity with the response phenomena being simulated and a good understanding of the mathematical models available to describe them.

3) Postprocessing and interpretation of the predictions of the computational model.

4) Utilization of the computational model in the analysis and design of engineering structures.

Within this general framework, CSM is being used today in a broad range of practical applications. To date, large structural calculations are performed that account for complicated geometry, complex loading history, and material behavior. Typical examples of structural calculations made in aerospace, automotive, off-shore oil, and nuclear industries are shown in Figs. 1-5.

Major advances in CSM continue to take place on a broad front. The new advances are manifested by the development of sophisticated computational models to simulate mechanical, thermal, and electromagnetic material behavior, efficient discretization techniques, computational strategies and numerical algorithms, as well as versatile and powerful software systems for structural analysis and design. Despite all the advances made, the detailed stress analysis and design of complex structures is very time consuming and, therefore, not economically feasible.

Structural and solid mechanics were the primary focus of computational mechanics up to the 1970's. However, during the 1970's and early 1980's, the emphasis of computational mechanics shifted to other mechanics disciplines, particularly, fluid dynamics. To date, there is a resurgence of interest in CSM. There are a number of compelling motivations for vigorously developing CSM. The first compelling motivation is that a number of unsolved current practical problems awaits experimental and/or numerical solutions. Some of these problems involve numerical simulations so large and complex that they overtax the capacity of even present-day large computers. Examples of these problems are the simulation of the response of transportation vehicles to multidirectional crash impact forces, dynamics of large flexible structures incorporating the effects of joint nonlinearities and nonproportional damping, and the study of thermoviscoplastic response of structural components used in advanced propulsion systems. Moreover, future flight vehicles (e.g., the national aerospace plane) are likely to require more sophisticated models than has heretofore been done, with a consequent increase in the computational cost of their analysis and design. This is because of the requirements of high performance, light weight, safety, and economy, and the associated stringent design criteria. In

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Received Nov. 18, 1986; revision received Feb. 5, 1987. 

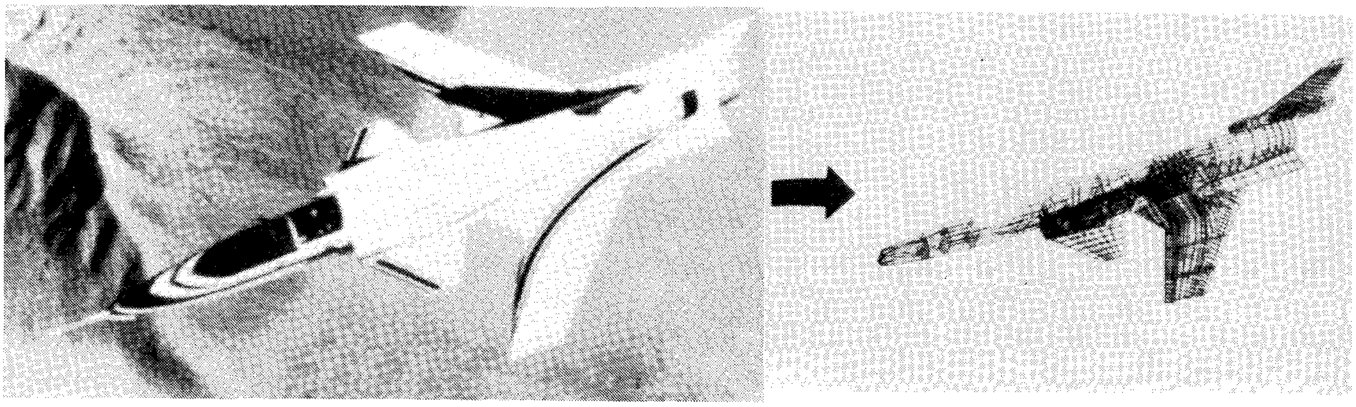

Fig. 1 X-29 forward-swept-wing aircraft demonstrator and its computational model. (Courtesy of Grumman Aircraft Systems, Bethpage, NY.)

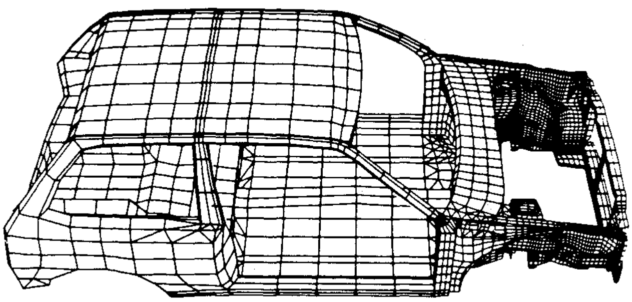

Fig. 2 Computational model of a car.[77]

other structural problems, the fundamental mechanics concepts are still being explored (e.g., in metal-forming problems, adequate characterization of the finite strain inelastic material behavior is needed). The application of CSM to these problems opens new technological approaches and provides insights into some aspects of the behavior that are difficult, if not impossible, to gain by alternate approaches.

A second compelling reason is that computer simulation is needed to reduce the dependence on extensive and expensive testing, which is frequently component- or mission-oriented. Moreover, in some mission critical areas in space, computer modeling may, of necessity, replace tests. This is because future large space structures (e.g., large antennas, large solar arrays, and the space station) are likely to exceed the limits of ground-test technology. The large size of these structures, their low natural frequencies, light weight, and the presence of many joints combine to defy confident testing in 1-*g* environment.

A third major motivation for developing CSM relates to the anticipated power and potential of emerging and future computer systems in solving large-scale structural problems. The potential of these future computers can be realized only by developing new formulations, computational strategies, and numerical algorithms that exploit the capabilities of these new machines [e.g., parallelism, vectorization, and artificial intelligence (AI) capabilities]. The development of such computational strategies and numerical algorithms is taking place in other fields of computational mechanics; notably, fluid dynamics.[228]

The importance of CSM to our national economy is reflected in the increased funding of different government agencies to CSM research. The Directorate for Structures at NASA Langley started a new initiative on CSM in October 1984. A similar program focused on advanced propulsion systems has also been under way at NASA Lewis. The two activities were then expanded to a joint program between NASA Langley and NASA Lewis the following year. The broad objectives of the NASA-CSM activity are to develop advanced structural analysis technology and to exploit modern and emerging computers in structures calculations. A description of this activity is given in Refs. 41 and 101.

The number of publications on computational structural mechanics has been increasing exponentially, and the literature on the subject is nearly overwhelming. Therefore, there is a need to broaden awareness among practicing engineers and research workers about recent developments in various aspects of computational structural mechanics. A number of survey papers and monographs have been written in the last few years on the advances made in some aspects of computational mechanics and CSM (see, for example, Refs. 32, 58, 90, 91, 129, 144, 167, 191, 215, and 237). Also, a number of symposia have been devoted to CSM and proceedings have been published (see, for example, Refs. 13, 38, 97, 109, 131, 143, 151, 154, 169, 225, and 233). However, there is a need for a state-of-the-art survey and an assessment of the different aspects of CSM. The present paper is an attempt to fill this void. Specifically, the objectives of this paper are: 1) to discuss key structural analysis and design requirements placed on CSM and computer technology, 2) to review and assess some of the major developments in CSM, and 3) to identify the future research areas in CSM that have high potential for meeting future technological needs. These are the items that pace the progress of CSM. The topics selected here are the ones that are of interest to the authors. The discussion is intended to give the structural analysts and designers some insight into the potential of the new advances in CSM for solving complex structural problems and to stimulate research and development of the necessary tools to realize this potential.

## Computational Needs for Future Structures Technology

The driving forces for future developments in CSM will continue to be: 1) the need for improved productivity and cost-effective engineering systems, and 2) support of innovative high-tech industries (aerospace related, transportation, petroleum, nuclear energy, shipbuilding, and microelectronics). In the aerospace field, planned future vehicles include the National Aerospace Plane, improved orbital delivery systems (with large payloads, low cost, and high reliability), structures subjected to very high accelerations, and very high precision shaped and controlled space structures under dynamic and thermal disturbances. The realization of the future aerospace systems requires technology advances in CSM as well as in a number of other disciplines, including materials, propulsion, aerodynamics, controls, avionics, optics, and acoustics. Similar advances are needed for the realization of the structures of future automotive, nuclear, and microelectronic systems. Among the technical needs for future high-performance structures are the

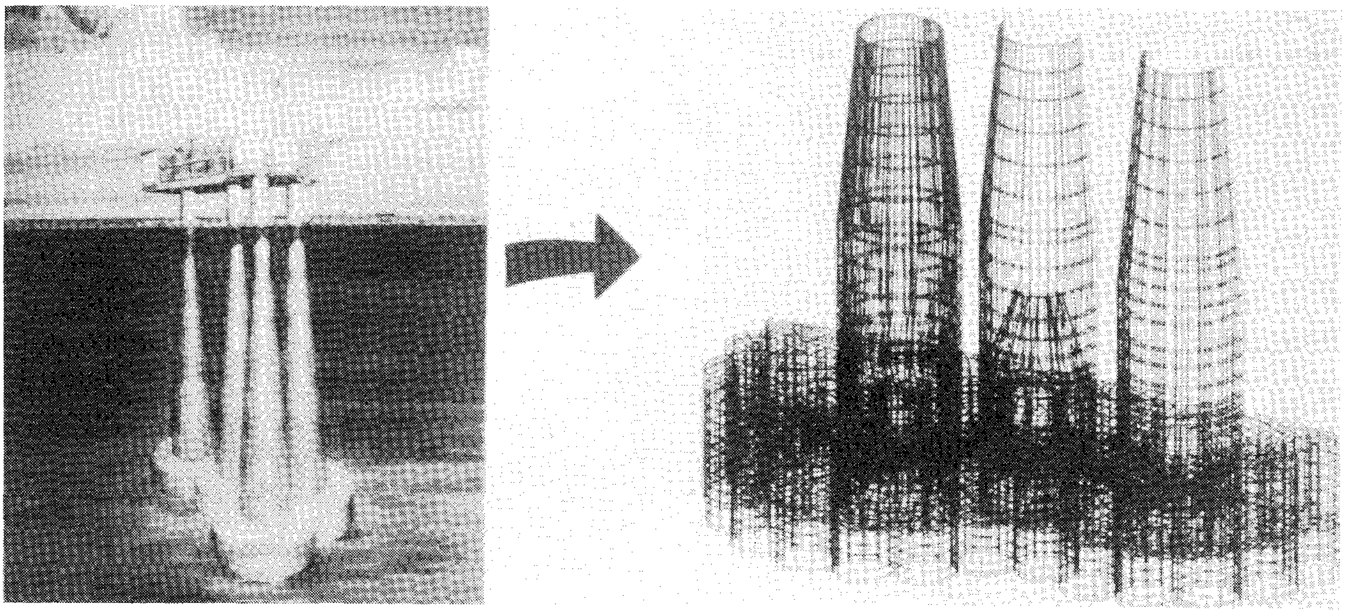

Fig. 3 Concrete oil platform and its computational model. (Courtesy of Veritec, Oslo, Norway).

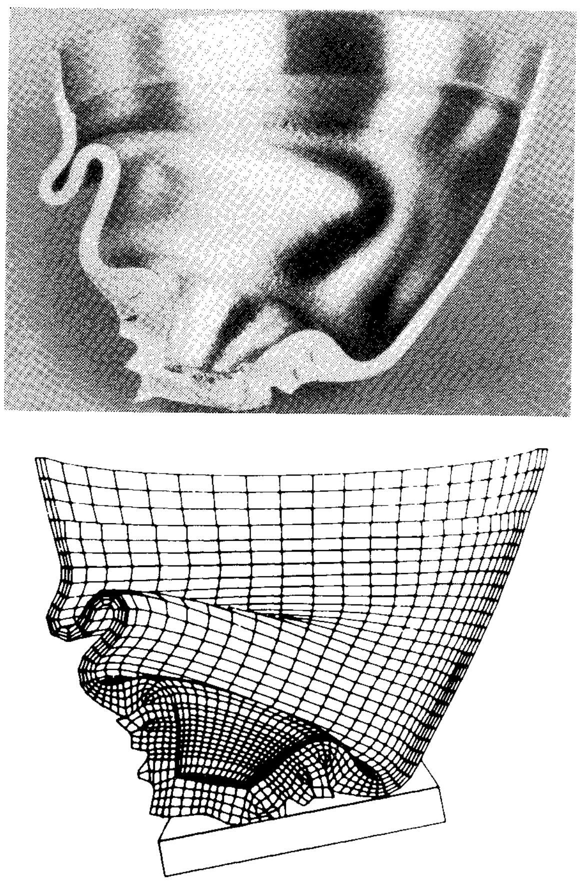

Fig. 4 Impacted nose-cone experiment on the top and numerical simulation on the bottom. (Courtesy of CRAY Research).

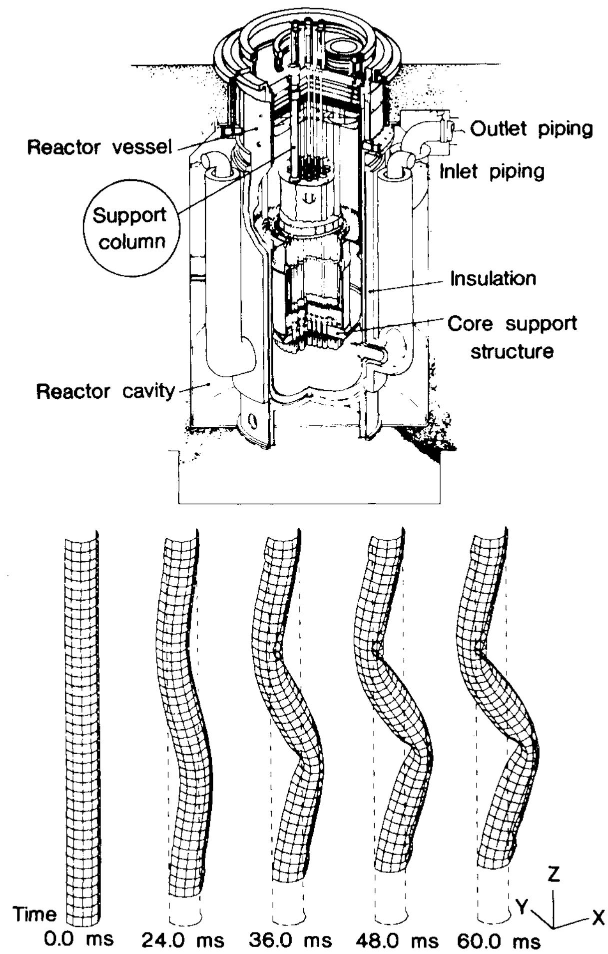


Fig. 5 Numerical simulation of the failure of a support column of the above core structure (ACS) of Liquid Metal Reactor (LMR).[98]

following:

1) Expanding the scope of engineering problems considered. This includes:

a) Examination of more complex phenomena (e.g., damage tolerance of structural components made of new material systems);

b) Study of the mechanics of high-performance modern materials, such as metal-matrix composites and high-temperature ceramic composites;

c) Constitutive modeling of new materials in the regime of large-strain plasticity, high-temperature creep, and high-strain-rate inelasticity;

d) Study of structure/media interaction phenomena (as would be required in the hydrodynamic/structural coupling in deepsea mining, the thermal/control/structural coupling in space exploration, the material/aerodynamic/structural coupling in composite wing design, and the electromagnetic/thermal/structural coupling in microelectronic devices);

e) More extensive use of stochastic models to account for uncertainties associated with loads, environment, and material variability;

f) Development of efficient high-frequency nonlinear dynamic modeling capabilities (with applications to impulsive loading, high-energy impact, structural penetration, and vehicle crashworthiness);

g) Improved representation of structural details, such as damping and flexible hysteritic joints;

h) Development of reliable life-prediction methodology for structures made of new materials, such as stochastic mechanisms of fatigue, etc.;

i) Analysis and design of intelligent structures with active and/or passive adaptive control of dynamic deformations. Potential applications of these structures include flight vehicles, large space structures, earthquake-resistant structures, as well as structures used in oil pipelines and geological drilling; and

j) Computer simulation (modeling) of the manufacturing processes of modern materials such as solidification, interface mechanics, and superplastic forming.

2) Development of a hierarchy of models, algorithms, and procedures for structural systems. Simplified and specialized models and algorithms are appropriate for use in the preliminary and conceptual design phases and more sophisticated models are used in the detailed design phase.

3) Development of practical measures for assessing the reliability of the computational models, and estimating the errors in the predictions of the major response quantities.

4) Continued reduction of cost and/or time for obtaining solutions to engineering design/analysis problems.

The hardware and software requirements to meet the aforementioned needs include:

1) Distributed computing environment encompassing high-performance computers (supercomputers) for large-scale structural calculations, and a wide variety of intelligent engineering workstations for interactive user interface/control and moderate-scale calculations (e.g., interactive model generation, evaluation of results in graphic form, and generation of elemental matrices). The organization of this hardware will require:

a) Extensive facilities for local and long-range networking to make all hardware and attendant software readily available to each user.

b) Development of demand-sensitive load-sharing systems to allow programs to migrate from one hardware/operating environment to another. Local networking can facilitate cooperative multidisciplinary investigations and design projects among team members interacting through shared databases. Inexpensive long-range networking can have several significant effects. Issues of proprietary data can be reduced by sending the program to the location of the owner of the proprietary data, performing the computations there, and receiving only the processed nonproprietary results.

2) User-friendly hardware interface or engineering workstations in the following capabilities: high-resolution and high-speed graphics, high-speed long-distance communication, and verbal (audio) as well as visual interfaces.

3) Artificial intelligence-based expert systems, incorporating the experience and expertise of practitioners, to aid in the modeling of the structure, the adaptive refinement of the model, and the selection of the appropriate algorithm and procedure used in the solution.

4) Computerized symbolic manipulation capability to reduce the tedium of analytic calculations and increase their reliability. This allows the analytic work to be pushed further before the numerical computations start.

5) Turnkey engineering application software systems that have advanced modeling and analysis capabilities and are easy to learn and use.

## Some Recent Advances in CSM

The development of CSM cuts across a number of disciplines including: 1) structural mechanics, 2) discretization techniques, 3) numerical analysis, and 4) computer science. Advances in each of these disciplines can have a strong impact on CSM. Some of the recent advances in material modeling and characterization, discrete element technology, numerical algorithms, computational strategies, and optimization techniques, which have had, or promise to have, a strong impact on CSM, are listed subsequently. The impact of new computing systems on CSM is discussed in the following section. The list of recent advances given herein is by no means complete or exhaustive; the intention is to stimulate the interest of researchers for realizing the full potential of these advances in the analysis and design of structures. Among the advances not fully covered herein are: computational methods in fracture and damage mechanics, effective computational strategies for coupled problems, computational methods for inverse problems (e.g., parameter/system identification), and trends in structural software systems. Recent developments and current research in the first two areas are contained in two review papers,[18,120] a monograph,[20] and in three symposia proceedings.[108,116,220] Trends in structural analysis software systems are discussed in Refs. 75 and 213.

#### Computational Models for Material Behavior

The reliability of the predictions of nonlinear structural response is critically dependent on the accurate constitutive modeling of the material behavior. Until recently, simple classical constitutive models were used to simulate the material behavior. For example, linear isotropic and linear kinematic hardening models were used to characterize rate-independent plastic response. These simple material models were found to be inadequate in a number of engineering applications, especially those involving large inelastic deformation and failure. Recent work on material modeling can be grouped in two general areas:

1) development of elaborate phenomenological models to simulate the small and finite-strain plastic material response; and

2) establishment of constitutive relations based on microscopic material structure and on micromechanical models.

The first research activity includes the development of phenomenological models to account for experimentally observed small-strain, cyclic plasticity phenomena,[222-224] and the generalization of small-strain linear kinematic hardening plasticity to the case of finite strains. The latter was, until recently, accomplished through the replacement of the strain increment and stress increment of the small-strain theory by the velocity strain, and the Jaumann rate of Kirchhoff stress, respectively, in the finite-strain theory. The anomalous behavior predicted by using this model in the simple-shear finite-strain case has been the subject of much discussion.[17,126] Several attempts are currently being made to develop a consistent finite-strain phenomenological theory of plasticity through the use of general isotropic tensor function expansions of the internal variables (e.g., back stress and plastic spin; see Refs. 62, 93, 107, 114, and 187). Extensions of this theory to the high-strain-rate and high-temperature regimes are also under investigation. Despite these studies, the subject of constitutive modeling of finite-strain plasticity is still unsettled.

Two important aspects of computational inelasticity that received attention in recent years are:

1) stability of numerical computations for strain-softening problems; and

2) the notion of objective integration.

Strain-softening computations require special provisions in the incremental/iterative solution strategy in order to survive; on the one hand, material instabilities and capture, on the other hand, bifurcation of the solution path (within load increments that are truly finite).[226] Objective integration includes the integration of constitutive relations to determine the stress from a given strain history, and the time integration in large deformation transient problems.[89,186,194]

The second research activity includes the extensive work devoted to the establishment of constitutive relations based on microscopic material structure and on micromechanical models for predicting the behavior and failure characteristics of materials (see, for example, Refs. 48, 129–131, and 225). These models attempt to quantify the physical features that underlie macroscopic material behavior and have been applied to traditional materials (metals and concrete) as well as to heterogeneous systems (e.g., new composites, high-temperature ceramics, and superalloys).

The material modeling studies were greatly aided by:

1) advances in microscopic experimentation and other nondestructive evaluation techniques (e.g., laser scattering), which allowed the observation of phenomena on the nanometer scale; and

2) the application of supercomputers to the modeling of material behavior on the microscopic scale. In fact, computer simulation is now recognized as an indispensable part of materials science.[6]

Among the recent significant contributions of the material modeling activity are:

1) Formulation and use of constitutive relations incorporating modeling of physical failure mechanisms (e.g., void nucleation and growth, and microcracking in metals; fiber debonding, cracking, and microbuckling in composites).

2) Analysis of failure mechanisms in terms of measurable and controllable parameters. This includes quantitative analysis of highly localized failure modes that are important precursors to ductile fracture in solids.

3) Use of computer simulation to provide fundamental understanding of some aspects of mechanical behavior that are difficult, if not impossible, to obtain by alternate approaches (e.g., the implications of various microstructural mechanisms of inelastic deformation on the macroscopic response).

The maturation of this activity will result in: 1) the development of a continuum of material models (and constitutive theories) for the linking and evolution of response phenomena across many levels of material and geometric scales (e.g., from the microscopic level to the structural level); and 2) the use of these models for the accurate prediction of the response as well as the failure of structural components.

### Advances in Discrete Element Technology

The establishment of reliable and efficient discrete elements for modeling structures with complicated geometry has been, and continues to be, the focus of intense research effort. In this section, some of the recent advances are reviewed. These advances are grouped into four categories: 1) formulative aspects of finite elements, 2) curved shell models, 3) singular and special finite elements for fracture mechanics applications, and 4) boundary element methods.

#### *Formulative Aspects of Finite Elements*

Although most of the finite-element models in use today are based on single-field variational principles (viz., displacement models), the past 10 years have witnessed an increasing interest in the use of alternate multifield (mixed and hybrid) formulations (see, for example, Refs. 11 and 207). This activity was motivated by the need for element models that are simple, have good predictive capability without any pathologies (such as spurious zero-energy modes), and whose performance is insensitive to distortion.

Multifield finite-element models can be developed through: 1) the use of multifield symmetric variational statements in which the displacement, stress, and/or strain fields are treated as independent fields; or 2) a priori relaxing the compatibility of displacements and/or the traction reciprocity at interelement boundaries in the variational statement. The latter approach requires the introduction of additional independent fields of Lagrange multipliers (tractions and/or displacements at interelement boundaries) to enforce the interelement constraints. A comprehensive summary of such multifield finite-element models is given in Refs. 7, 8, 10–12, 145, and 230. Efficient three-field mixed models based on the Hu-Washizu principle were proposed in Refs. 35 and 128.

Among the advantages of multifield finite-element models over the single-field models are the following[145,178,182,234]:

1) Simplicity of element formulation. This is particularly true for nonlinear problems and for problems in which the single-field formulation requires $C^1$ continuity (e.g., thin-plate and shell problems). The corresponding multifield formulation requires only $C^0$ continuity.

2) Higher accuracy for the different fields. This is particularly true for elastoplastic and fracture mechanics problems.[14,125,186,188] The higher accuracy is more pronounced when single-field models are based on incomplete polynomial interpolation functions for the displacements.

3) Overcoming the locking problems in constrained media applications. Single-field finite-element models can encounter computational difficulties or exhibit locking phenomena (due to overestimating some of the stiffnesses) in constrained media applications (such as incompressible solids and shear flexible thin plates and shells). Several remedial actions have been proposed for use in conjunction with single-field models. These include the use of reduced/selective integration, mode-decomposition projection methods, assumed natural strain methods, and penalty methods. Multifield models automatically overcome these problems. The equivalence between the single-field finite elements based on these approaches and the elements based on multifield variational principles has been established in Refs. 88, 118, 138, 142, and 202.

While the aforementioned advantages of multifield finite-element models have been documented, and the mathematical basis of their stability and convergence have been established in Refs. 22, 46, and 47, there are a number of drawbacks associated with these models, including

a) the presence of additional degrees of freedom, which result in a larger system of indefinite equations;

b) the presence of spurious, zero-energy modes in some of the efficient multifield models;

c) the difficulty of developing curvilinear multifield elements that possess the properties of observer invariance and objectivity, and satisfy the patch test; and

d) the lack of a simple approach for selecting the trial functions for the different fields in order to satisfy the Ladyszhenskaya-Babuska-Brezzi (LBB) conditions.

A general discussion of some of these drawbacks is given in Refs. 230 and 231.

Two remedial actions have been proposed to overcome the first drawback, namely; 1) using discontinuous bases for the stress and strain variables and eliminating these variables on the element level, and 2) solving the mixed system of equations iteratively. The first approach is used in Refs. 138 and 142. The second approach is presented in Ref. 128, in which an augmented Hu-Washizu variational principle is used to construct a family of iterative algorithms.

A number of approaches have been proposed for the removal of the spurious modes and for element stabilization (see, for example, Refs. 27, 33, 110, and 175). Also, removal of the spurious modes on the global (structure) level has been suggested in Refs. 94 and 199. This can be accomplished by postprocessing of solutions or by application of supplemental forces to the lines of nodes along the boundary to eliminate global instabilities. Removal of the spurious modes on the global level has been shown to be computationally more efficient (than element stabilization) and to have little effect on the physical solution.

To maintain the objectivity of the element stiffness matrix in a mixed-hybrid formulation, the stress/strain tensor components need to be assumed in an element-local coordinate system (as functions of the element natural coordinates) and not in a global coordinate system.[181,182,195] The theory of symmetry groups provides a useful tool in choosing the least-order stress/strain fields that lead to an element stiffness matrix which is of the correct rank (i.e., with no spurious modes) and is also objective. Much remains to be done to fully exploit the potential of multifield models in complex structural problems.

*Curved Shell Elements*

The development of efficient finite elements for modeling curved shell structures has been, and continues to be, an active research area. Some of the advances made in this area are contained in a recent monograph[91] and a review paper.[36] Four recent contributions are worth mentioning. In the first, a mixed interpolation is used for the various strain tensor components to generate simple elements free of spurious modes. The bending and extensional strain components are calculated as usual from the displacement interpolations, while the transverse shear strain components are interpolated differently. Applications of these elements to linear and nonlinear problems are reported in Refs. 29, 30, and 65. The second contribution is the development of free-formulation shell elements with six degrees of freedom per node. The formulation allows the selection of nonconforming (and higher-order) trial functions. Convergence is guaranteed through the satisfaction of the "individual element test," which implies the satisfaction of the patch test. The degrees of freedom are the usual three translational, the two in-plane rotational freedoms per node, and the so-called "drilling rotional freedom." The drilling freedom eliminates the singularity associated with coplanar elements at a node. The high accuracy obtained by using these elements for nonlinear problems is demonstrated in Ref. 39. The third noteworthy contribution is the development of continuum-based resultant shell elements for nonlinear analysis, based on mild assumptions regarding the magnitude of transverse shear deformation and the use of resultant quantities as primitive variables, thereby avoiding the numerical integration in the thickness direction. For multilayered (or inelastic) shells, this can lead to considerable savings over the continuum-based (degenerated) shell formulations.[208] The fourth noteworthy contribution is the development of an element-independent corotational procedure for the treatment of arbitrarily large rotations.[184]

*Singular and Special Finite Elements for Fracture Mechanics Applications*

In the analysis of the safety of flawed structures using fracture mechanics techniques, two types of singularity play a major role. The first is a point singularity, which is defined as a singularity of the first derivative of the solution at a point. The second is a line singularity, which denotes a singularity at each point along a curve in three-dimensional space.

Examples of the first type are singularities at reentrant corners, tips of sharp cracks in two-dimensional domains, and vertices (where three or more planes may intersect) in three-dimensional domains. An example of the second type is the singularity in the displacement derivatives (or strains in a linear theory) near the border of an arbitrarily shaped flaw (surface of discontinuity) in a three-dimensional solid. If $r$ is the distance from the point of singularity (measured in a plane normal to the line), the solution function behaves as $r^{\lambda}$, where $\lambda$ may, in general, be complex, and the real part of $\lambda$ is less than unity.

A number of special and singular elements have been developed in which the field variable varies as $r^{\lambda}$ and the derivative of the field variable has a singularity of the type $r^{\lambda-1}$. A comprehensive survey of these singular elements is given in Ref. 14.

For three-dimensional problems of embedded or exposed elliptical cracks, a Schwartz-Neumann-type alternating method, based on the use of an ordinary finite-element method for analyzing the uncracked solid, and an analytical solution for an embedded elliptical crack was found to be an efficient alternative to a singular finite-element method.[16,134]

*Boundary Element Methods*

The boundary element method was introduced in the early 1970s as an application of the finite-element technology to the boundary integral equation formulation. Recently, it has emerged as a powerful tool for solving structural and solid mechanics problems. Since the method requires the modeling of only the boundary rather than the entire domain of the problem, the dimensionality is reduced and the input preparation is considerably simplified.

Since the introduction of the boundary element method, a great expansion of the technology and applications associated with the method have taken place. These included nonlinear, transient, harmonic response calculations as well as design applications. Also, a number of computer programs are available for commercial use, including a program currently being developed for transient thermoviscoplastic analysis of three-dimensional solids (BEST—Boundary Element Solution Technique) with application to gas turbine structures. Reviews of some of these applications are contained in Ref. 72 and in the proceedings of two recent symposia.[55,61]

For boundary value/initial value problems in solid mechanics, it is often possible to derive certain boundary integral representations for displacement through the use of unsymmetric variational statements in which the test functions are required to be differentiable to a higher degree than the trial functions.[9,15,44] Examples of boundary element methods derived through the discretization of such unsymmetric variational statements are given in Refs. 9, 40, and 210.

For a number of linear and nonlinear problems, the integral representation involves not only boundary integrals, but also integrals over the interior of the domain of the trial functions and/or their derivatives. Examples of these problems are: 1) anisotropic and nonhomogeneous solids, 2) linear problems of infinite domains (unbounded media) in which the fundamental solution cannot be established for the entire linear operator, and 3) nonlinear problems involving large deformation and material inelasticity. In all of the problems cited, discretization of the integral representations lead to the field boundary element method.[15]

Recent studies have demonstrated that the boundary element method and the field boundary element method offer significant computational advantages over the traditional finite-element methods in the following situations:

1) linear problems of infinite domains (unbounded media) in which the fundamental solution can be established for the entire differential equation[45];

2) analysis of structures with material nonlinearities, particularly in the presence of cracks and high-stress gradients (or singularities);

3) linear and nonlinear thin-plate and shell problems. Although the classical boundary element method was found to be not competitive with the finite-element method for these problems,[95,96] the field boundary element method is quite competitive.[235,236] The trial functions for the transverse

displacement need not satisfy any continuity requirements; and

4) problems of control of the nonlinear dynamic response of large structural systems.[21]

For a number of practical problems, the simultaneous application of the boundary element method and other discretization techniques (e.g., finite-element method) is more effective than the use of either one of the techniques.[9,45] This can be expected to be an area of extensive research and development in the future.

### Computational Strategies for Nonlinear Static Problems

Considerable progress has been made in the development of computational strategies for nonlinear static and postbuckling problems. For the sake of the present discussion, it is useful to distinguish between two classes of problems: problems with smooth and rough nonlinearities.[68] Recent advances made in the development of computational strategies for both classes of problems are outlined subsequently. The first class is characterized by smooth relations at the local level and by the fact that the nonlinearities are reversible. Examples of these problems are geometrically nonlinear structural problems, nonlinear elasticity problems, and follower-force problems. Note that the overall structural behavior in these problems may not be smooth, as witnessed by the phenomena of buckling, snapping, and flutter.

Problems with rough nonlinearities are characterized by discontinuous field equations, involving inequality constraints and path-dependent nonsmooth local response. Examples of these problems are flow-rule plasticity problems and frictional contact problems. The frictional contact problems are discussed in a succeeding subsection.

Recent work on strategies for problems with smooth nonlinearities has focused on development of incremental quasi-Newton methods with efficient Jacobian updating (e.g., Broyden-Fletcher-Goldfarb-Shanno method, see Refs. 64 and 212); techniques for determining the critical (singular) points in the equilibrium path (bifurcation and limit points) and tracing the postcritical point paths; and efficient methods for handling large rotations. Among the noteworthy contributions are the developments of:

1) dynamic relaxation techniques[67,216,219] and constraint methodologies (e.g., arc-length techniques) for circumventing the numerical problems at critical points (see Refs. 60, 171, 172, 189, 192, and 203);

2) a matrix formulation of compound finite rotations[5] and an element-independent corotational procedure for the treatment of arbitrarily large rotations[184];

3) large-strain inelastic analysis in a consistent natural formulation, which encompasses all of the thermodynamic aspects[4];

4) reduction methods by which considerable reduction can be made in the total number of degrees of freedom used in the initial discretization for bifurcation buckling, postbuckling, and nonlinear analyses. These reduction methods are discussed in a succeeding subsection.

The numerical ability to solve postbuckling and nonlinear inelastic problems with a moderate number of degrees of freedom has advanced beyond the ability to characterize the nonlinear material response. This is particularly true for finite-deformation and rate-dependent plasticity problems (problems with rough nonlinearities).

The implementation of current results from modern mathematical analysis into computational procedures can significantly enhance the understanding and the description of instability processes of complex structural systems.

### Transient Response Analysis

The computer simulation of transient response of structures has received considerable attention in recent years. The drivers of this activity included the reliable simulation of automotive and aircraft crash phenomena, behavior of structures subjected to conventional and nuclear weapons effects, safety studies of nuclear reactors in hypothetical accidents, ground motion and behavior of structures during earthquakes, numerical simulation of high-velocity impact phenomena (including material response to penetration processes), study of dynamic deformation of materials associated with high-speed metal forming or machining, and deployment dynamics and control of large flexible space structures. The modeling of these mechanical phenomena often requires temporal integration of the governing equations and the treatment of diverse nonlinearities.

Reviews of computational methods used in penetration mechanics and constrained dynamical systems are contained in Refs. 52, 132, and 240. Computational methods for contact/impact problems are discussed in the succeeding subsection. In this section, a brief account is given of the recent advances in temporal integration techniques.

Until the mid-1970s, the temporal integration techniques were segregated into two distinct classes: explicit and implicit. Explicit schemes involve no matrix equations, and are felt to be cost effective for the analysis of wave propagation phenomena in continua. Their shortcoming is that numerical stability considerations dictate the use of small time steps, often smaller than necessary for accuracy. Implicit algorithms, on the other hand, require solution of matrix equations during each time step. Their major advantage over explicit schemes is the improvement in numerical stability. They are believed to be cost effective in cases where the response is dominated by the low modes.

In recent years, various improvements which resulted in considerable reduction of the analysis time and cost have been reported in temporal integration schemes. Also, temporal integration schemes for new multiprocessor computer systems have been developed.[163,168] Among the new techniques are partitioned schemes in which concurrent use is made of both implicit and explicit temporal integration for different spatial domains, or in alternating temporal sequence, alternating direction methods, staggering schemes, domain decomposition and fractional/subcycling time-marching techniques in which subdomains of the mesh are integrated at different time steps. Some of these advances are summarized in Refs. 32, 34, 109, and 173). Also, a review of temporal finite-element methods, derivable from the Hamilton weak formulation in the time domain, is given in Ref. 239.

Other methods that combine the attractive features of explicit and implicit techniques include operator splitting techniques.[174,179,218] In these methods, an implicit integrator is generally chosen as a starting point, and the matrix operator of the equations is split so that solution of equations is avoided or the system to be solved is reduced in size. The motivation for this procedure is to retain the stability of the implicit integrator and achieve the computational efficiency of an explicit integrator.

Although progress has been made, the solution of large-scale nonlinear dynamic problems is still not economically feasible on present-day large computers. Moreover, temporal integration modules in many existing commercial programs still lack robustness and user-friendliness. It is anticipated that considerable further activity will be seen.

### Computational Methods for Contact-Impact Problems

Many important problems involve contact, impact, and/or frictional sliding between two or more structural components. Examples are problems emanating from joint design, tire contact, vehicle safety, crashworthiness, and weapons technology. The major issue in the computer simulation of contact/impact phenomena is the absence of a physically and mathematically sound model of the phenomena. Moreover, in the presence of friction, uniqueness of solution cannot be generally guaranteed (see for example, Refs. 100 and 165). In the cited references, some of the computational difficulties encountered in the solution of dynamic contact problems are described.

Recently, a number of efficient algorithms have been developed for solving some contact/impact problems involving sticking; frictional sliding and separation between solid bodies. These algorithms are generally based on either the Lagrange multiplier or penalty methods. Applications of these algorithms to the problems of contact of solid bodies and tire contact are presented in Refs. 162, 166, 204, 229, and 232. Also, general-purpose programs have been developed for the solution of large-deformation contact/impact problems of three-dimensional solids. Those include the DYNA3D explicit code and the NIKE3D implicit code.[84,85] For large-deformation problems, the use of an arbitrary Lagrangian-Eulerian (ALE) description of the motion was found to be more advantageous than either the Lagrangian or Eulerian descriptions.[80] In most of the cited applications, Coulomb's law of friction is used in the evaluation of the tangential tractions from the normal tractions. However, in some of the studies, a nonlocal friction law is proposed (see, for example, Ref. 164).

A noteworthy contribution in the study of contact of solid bodies using phenomenological friction models is given in Refs. 119 and 165. In the cited references, a separate constitutive relation is assigned to the interface, independent of those characterizing the parent solids. For a large range of frictional phenomena, a semiempirical, power-law-type constitutive relation for the interface, based on data obtained from a number of experiments, is shown to be adequate.

## Hybrid Analysis Techniques

Recent experience with the commonly used approximate analysis techniques has shown that improving the efficiency of the numerical algorithms used with these techniques results only in a marginal reduction in the computational cost of the analysis. On the other hand, the application of a hybrid combination of approximate techniques to the analysis of a complex structure can result in a dramatic saving in computational effort. Many situations can be cited where hybridization resulted in substantial improvement of the efficiency of the resulting system. In the area of numerical algorithms, the use of a hybrid direct/iterative technique for the solution of the algebraic equations of the multigrid finite-difference method (described in the succeeding section) or the hierarchical finite-element method was shown to be more effective than the use of either one of these two techniques. In structure/media interaction problems, the application of combined explicit/implicit temporal integration schemes can be considerably more efficient than the application of the individual schemes. Even in the material science area, the hybrid combination of different materials can result in new material systems with properties better than those of the parent materials. In this section, recent advances in three groups of hybrid techniques are discussed. The three groups are: 1) hybrid analytical techniques, 2) reduction methods, and 3) hybrid numerical/experimental techniques. Other hybrid analysis/modeling techniques that have high potential for structural problems include the various global-local strategies, the hybrid spectral-numerical discretization techniques (e.g., spectral element methods), and the combined finite-element/field boundary element method. The global-local methodologies have high potential for predicting the detailed stress distribution in complex structures. An assessment of the effectiveness of using these techniques in nonlinear and postbuckling analysis is given in Ref. 153. Spectral element methods exploit the common weighted-residual foundation of spectral and finite-element techniques, and combine advantages of both parent techniques; namely, the rapid convergence of spectral methods and the versatility of finite-element methods. For a discussion of these techniques, see Ref. 176.

### *Hybrid Analytical Techniques*

Approximate analytical techniques such as the perturbation method, the Bubnov-Galerkin and Rayleigh-Ritz techniques were standard tools for the analysis of structures prior to the advent of digital computers. These analytical techniques have the major advantage over numerical discretization techniques (such as the finite-element method) of providing physical insight into the nature of the response. Moreover, analytical techniques can be used in conjunction with partitioning schemes for the analysis of individual components of practical (complex) structures. The resulting method is referred to as the global (or macro) element method. The widespread availability of computers and the fascination with the finite-element method, caused by its versatility in handling complex structures, and simplicity of computer implementation, has resulted in a relative stagnation in the development of effective analytical techniques.

Recently, a hybrid analytical technique that combines both the standard regular perturbation method and the classical direct variational technique was developed and applied to the solution of steady-state nonlinear thermal problems and to geometrically nonlinear structural problems.[146,150] This hybrid technique is basically a two-step technique. The first step involves the generation of coordinate functions (or modes) for the fundamental unknowns using the standard regular perturbation method. The second step consists of computing the amplitudes of the coordinate functions (or modes) via a weighted residual method or a direct variational technique. The development of the hybrid technique was greatly aided by using the computerized symbolic manipulation system MACSYMA[121] in performing the analytical tasks (viz., generation of: 1) perturbation equations, 2) coordinate functions, and 3) the nonlinear algebraic equations in the amplitudes of the coordinate functions).

The hybrid technique was shown to extend the range of applicability of the perturbation method and enhances the effectiveness of both the weighted residual method and the direct variational technique. It also alleviates the following major drawbacks of the two classical techniques:

1) the requirement of using a small parameter in the regular perturbation expansion is avoided; and

2) the method provides a systematic selection of the coordinate functions (or modes) needed in the weighted residual method and the classical variational technique.

### *Reduction Methods*

The large numbers of degrees of freedom used in modeling complex structures are often dictated by the topology of these structures rather than by the expected complexity of the behavior. This fact has been recognized and techniques for reducing the degrees of freedom have long been proposed in vibration analysis and automated optimum design (see, for example, Refs. 71, 135, and 177), and more recently in nonlinear analysis. The techniques for reducing the degrees of freedom have been referred to as reduction methods.

Reduction methods are hybrid two-step techniques, based on the successive application of a discretization technique (finite elements, finite differences, boundary elements, or a combination of them) and a classical variational technique (Rayleigh-Ritz or Bubnov-Galerkin technique). The discretization technique is used to generate a few global approximation vectors (or modes). The classical variational technique is then used to compute the amplitudes of these modes. The primary objective of using reduction methods is to reduce considerably the number of degrees of freedom in the initial discretization and, hence, to reduce the computational effort involved in the solution of the problem. Successful applications of reduction methods to a variety of structural problems with moderate- and large-rotation geometric nonlinearities have been reported in Refs. 2, 136, 137, 139, 140, and 148. Application to pin-jointed trusses with combined geometric and material nonlinearities is reported in

Ref. 141. Application to thermal problems is reported in Ref. 149.

Two recent applications of reduction methods deserve further examination. In the first, only partial reduction is made. The degrees of freedom in the region of strong nonlinearity are retained, and the other degrees of freedom are reduced.[153] In the second application, the nonlinear response of a complex structure (e.g., anisotropic or stiffened panel) is generated using small (or large) perturbations from the response of a simpler system (e.g., orthotropic or unstiffened panel). It is also possible to use a hierarchy of simpler structural systems in generating the response of the original complex structure. This is accomplished by splitting the governing equations of the original structure, choosing a number of perturbation parameters, and successively applying a single-parameter reduction method with each of the parameters. Application of this strategy to the static and free-vibration analyses of shells of revolution is described in Refs. 155 and 158. The same strategy is used to reduce the size of the analysis model used in predicting the free vibration, bifurcation buckling, and nonlinear responses of symmetric anisotropic panels in Refs. 152 and 156, and the linear static response of symmetric structures with unsymmetric boundary conditions in Ref. 159.

*Hybrid Numerical/Experimental Techniques*

Although the application of hybrid experimental/numerical techniques to stress analysis problems dates back to the 1950s, it was not until the mid-1970s that the complementary role of numerical and experimental techniques was exploited. In a number of applications, the hybrid experimental/numerical technique yielded reliable information about the response that could not be obtained by the single use of either the experimental or numerical techniques. An example of these applications is to input the experimentally determined initial and boundary conditions in the numerical program, thereby improving the accuracy of the predicted response.

Other successful applications of hybrid experimental/numerical techniques are in the areas of structural crash prediction and dynamic fracture mechanics.[16,102,103,133,196] In the hybrid methods of structural crash prediction, the structure is divided into a number of relatively large sections or subassemblies modeled by using simple finite elements (e.g., beam/spring systems). The nonlinear deformation and crushing characteristics of these finite elements are obtained from static deformation (and crushing) tests, and can be tuned by adjusting the input data, to optimize the accuracy for a given impact case. This approach was often referred to as "hybrid modeling" and proved to be quite effective for the representation of an underfloor crushable structure of aircraft. A computer program (KRASH[227]) based on this hybrid modeling approach has been developed by Lockheed-California Company.

In the fracture mechanics area, the parameters which govern the elastic, elastic-plastic, and dynamic fracture (with the exception of geometric quantities such as crack-opening displacements and crack-tip opening angles) cannot be measured directly. The hybrid experimental/numerical technique provides computed fracture parameters (such as the $J$ integral) under actual test conditions, and not under assumed test conditions that are normally prescribed in pure computational methods.[102,103]

*Numerical Algorithms*

The success of discretization processes is dependent on the skillful use of efficient numerical algorithms in various phases of the analysis, including the handling of the resulting large systems of algebraic equations. Early commercial finite-element codes relied entirely on direct equation solvers (i.e., those based on the Gaussian elimination method). Many attempts have been made to improve the efficiency of direct solvers and to reduce the storage requirements. These efforts resulted in compacted column and frontal techniques, as well as in the hypermatrix storage scheme for very large problems. More recently, it has been established that iterative techniques, which avoid the formation and factorization of a global system of equations, are more efficient than direct solvers for many three-dimensional finite-element problems. The same is true when finite differences are used for the numerical discretization, because the regular structure of the equations is particularly well suited to iterative processes. Recent work on numerical algorithms has focused on developing efficient iterative techniques as well as numerical algorithms for exploiting the capabilities of new computing systems (parallel and vector computers). For a review of numerical algorithms developed for new computing systems, see Refs. 167, 197, and 228.

In this section, recent advances in two strategies for accelerating the convergence of iterative methods are discussed; namely, multigrid processes and preconditioned conjugate gradient techniques. Advances in a group of numerical algorithms that reduces the storage requirements for large-scale problems (viz., element-by-element techniques) are discussed in Ref. 92.

*Multigrid Processes*

In multigrid methods, a sequence of nested grids is used, and the solution process involves relaxation, transfer of residuals from fine to coarse grids, and interpolation of corrections from coarse to fine levels. Multigrid methods were first introduced into the finite-difference methods in the last decade as a means of accelerating the convergence of iterative processes and were hailed as one of the most significant developments there.[42,170] It was later established that standard, hierarchically formulated finite elements present essentially a multigrid structure without any special treatment.[59] Multigrid methods can generate the solution of a linear system of equations in $\mathcal{O}(n)$ arithmetic operations, where $n$ is the number of equations.

A basic difficulty of the classical multigrid methods is the assumption of nestedness of the grids (or of the shape functions),[81,123,124] which may lead to complications when analyzing realistic structures. To alleviate this drawback, two approaches were recently proposed.

The first approach is referred to as the algebraic multigrid (AMG) method and uses the matrix of the algebraic system of equations associated with the fine grid as input. No physical grids are constructed.[43]

The second approach uses sets of unnested grids and is referred to as the unstructured multigrid method. The unstructured multigrid solver utilizes an efficient intergrid interpolation procedure and then performs sequential passes over the grids in the multigrid fashion until convergence is achieved.[113]

Adaptation of multigrid processes to vector and parallel computers has been discussed in Ref. 122.

*Preconditioned Conjugate Gradient (PCG) Techniques*

The conjugate gradient iterative technique has received considerable attention in recent years. The rate of convergence of the conjugate gradient methods depends on the condition number of the global stiffness matrix: the smaller the condition number, the more rapid the convergence. Hence, much of the recent work on conjugate gradient methods has been devoted to the development of preconditioning strategies. Among the effective preconditioning strategies are the matrix splitting techniques (e.g., use of diagonal or block diagonal matrix as a preconditioner[54,78,99]). Matrix splitting techniques can be very efficient for analyzing anisotropic and quasisymmetric structures. The preconditioners for these structures can be selected to be the matrix of the corresponding orthotropic and symmetric structures, respectively. These particular choices of the preconditioner, in addition to expediting the convergence of the iterative process, provide sensitivity information on the response of the structure to the anisotropy and the unsymmetry of the structure.[160]

The conjugate gradient technique is particularly suited for vector computers. However, on some parallel computers, the inner products can slow down the computation. Adaptation of the PCG technique to parallel computers is discussed in Refs. 167 and 200.

### Quality Assessment and Control of Numerical Simulations of the Structural Response

Despite the significant advances made on the theories and algorithmic tools of discretization methods, the selection of the computational model for a particular problem is largely based on intuition and experience gained from solving similar problems. Much effort is now being directed towards assessing the accuracy of the numerical response predictions and improving their quality. Recent developments in this area are contained in a number of conference proceedings.[19,23,25] In this section, some of the recent advances made in assessing the reliability of the computational model and in postprocessing the numerical solutions to improve their quality are discussed.

#### *Reliability of the Computational Model*

The assessment of the reliability of the predictions of a computational model continues to be the most difficult aspect of the numerical simulation of the structural response. The term reliability is used herein to denote the agreement between the response predictions of the computational model and those of the actual structure. The reliability of the predictions of the computational model is influenced by the following factors[26]: 1) the reliability of the mathematical model selected for describing the response of the actual structure; 2) errors and uncertainties in the input information for the mathematical model (e.g., uncertainties in material properties, geometry, boundary conditions, and loading information); and 3) errors caused by the numerical discretization of the mathematical model, truncation of infinite processes (e.g., iterative processes and summation of infinite series), and the implementation of numerical algorithms on computers with finite precision.

An assessment of the reliability of the mathematical model requires the identification of the range of validity of the mathematical theory used in describing the structure. The effect of an improper selection of the mathematical model for simple, plane elasticity problems has been demonstrated in Ref. 26.

The uncertainties in the input information can be handled by using either[26]: 1) stochastic formulation in which the data and the response quantities are stochastic functions; 2) bracketing the response for an admissible range of the input data; or 3) assessing the effect of uncertainties by evaluating the sensitivity of the response of the structure to each of the input parameters.

The discretization error represents the difference between the exact and the approximate (e.g., finite element) solutions for the mathematical model. The assessment of discretization errors for practical structures requires the selection of an appropriate measure for the discretization error and the calculation of an a posteriori estimate for this error.

In recent years, considerable interest has been shown in the development of reliable error estimates as well as feedback procedures (or adaptive strategies) by which a required accuracy of the finite-element solution can be reached most economically. Also, considerable work has been devoted to the control of round-off errors and their accurate estimation. Review of some of the recent developments on quality assessment and control of finite-element solutions is contained in a recent survey paper.[157] An overview of the current state of the art of round-off error estimation is contained in Ref. 105.

Most of the a posteriori error estimates reported in the literature are for stationary (elliptic) problems and are based on either the interior and boundary residuals or the local energy norm. The first represents the equilibrium defects in the interior, on the portion of the boundary where tractions are prescribed, as well as the jumps in the tractions at interelement boundaries. The energy norm is defined as the square root of the strain energy of the error. Pointwise error estimates for detailed response quantities (e.g., stresses and displacements) are available only for simple problems (e.g., one-dimensional problems and the Saint-Venant torsion problem).

Adaptive improvement of finite-element solutions aims at improving the quality of the solutions by enriching the approximation in some manner so as to achieve the "best" solution for a given computational effort (or cost). Four adaptive strategies have been proposed in the literature.[157] These are: 1) successive selection of the meshes ($h$ method), 2) moving the nodes ($R$ method), 3) successive selection of the order of the polynomial approximation inside some elements ($p$ method), and 4) simultaneous selection of the meshes and the local order of the approximation ($h$-$p$) method.

For stationary (static) problems, the $h$, $p$, and $h$-$p$ methods are more commonly used than the $R$ method. For time-dependent (transient) problems, adaptive strategies are based on either the moving mesh techniques or the combination of mesh moving and refinement (or derefinement).

The mathematical theory for a posteriori error estimates for the discretization error and the adaptive processes is very incomplete and has been tested only on simple sets of problems.

#### *Postprocessing of Finite-Element Solutions to Improve Their Quality*

Often the primary objective of the finite-element analysis is to determine certain response quantities, such as stress intensity factor, stresses and strains in some area of the structure. Direct computation of this data from the finite-element solution can result in lower accuracy for the computed quantities (e.g., stresses computed from displacement finite-element models). Recent studies have demonstrated that the accuracy and rate of convergence of stresses depend on how (and where) they are computed. Several approaches have been suggested for improving the accuracy of stress calculations.[24,49,50,57,79,115,209] Several of the postprocessing approaches are based on the superconvergence concepts and include iterative refinement techniques for reducing the equilibrium errors. Some of these techniques were recently shown to be equivalent to mixed formulations in which the stresses and displacements are primary variables.[238]

### Stochastic-Based Modeling

Analysis and design methods for engineering structures are generally based on deterministic parameters. In practice, there are often uncertainties associated with material and geometric properties, external forces, and boundary conditions. Although, in most situations the uncertainties may be small, the combination of these uncertainties can lead to large and unexpected excursions of the response, particularly in multicomponent systems.

In the past, problems with uncertainties have been studied to provide an insight into the statistical response variations. The methods used in these studies were based on either a statistical or a nonstatistical approach. The first approach includes simulation (involving sampling and estimation). Among the most commonly used simulation techniques are direct Monte Carlo simulation, stratified sampling, and Latin hypercube sampling.[106,183,221] Nonstatistical approaches include numerical integration, second-moment analysis, and probabilistic finite-element methods.[3,56,76,111,117,127] For linear structural systems, second-moment analysis techniques proved to be effective. However, their application to nonlinear systems is still not feasible. Hence, much of the recent work has focused on probabilistic finite-element methods.

In probabilistic finite-element methods, the response quantities (stresses, strains, and displacements) are expressed as random variables governed by probability density functions. This allows the uncertainties associated with these response quantities to be treated in a quantitative but realistic manner. Probabilistic finite-element methods were recently formulated for linear and nonlinear continua with inhomogeneous random fields. The random field is discretized in a manner analogous to the discretization of the displacement field in single-field finite-element models[111,112] or the displacement and stress fields in multifield finite-element models.[70] New approaches are currently being developed to introduce probabilistic aspects of geometry, boundary conditions, material properties, loading conditions, and operational environment directly into the structural analysis formulation for realistic turbine blade models. This work is supported by NASA Lewis and is described in Ref. 51. Also, a state-of-the-art review of probabilistic methods for engineering analysis is given in Ref. 180.

The ability to quantify inherent uncertainties in the response of engineering structures is obviously a great advantage. But the principal benefit of using any stochastic method consists of the insights into engineering, safety, and economics that are gained in the process of arriving at those quantitative results and carrying out reliability analyses. It is likely that as structures become more complicated, failure mechanisms will be probalistically modeled from the beginning of the design process, and potential design improvements will be evaluated to assess their effects on reducing overall risk. The results, combined with economic data, will be used in sequential cost-benefit analyses (perhaps also done on a probabilistic basis) to determine the structural design with the most acceptable balance of cost and risk.

#### Techniques for Large-Scale Optimization Problems

As a result of the intensive research on structural optimization techniques, these techniques are gaining more acceptance in engineering practice, particularly in aerospace and automotive industries. A number of large-scale optimization/design systems have been developed and are used in industry. A review of 30 of these optimization/design systems is given in Ref. 87. Other large design systems are currently under development, including the ASTROS (Automated Structural Optimization System) project for airframes and space structures funded by the Flight Dynamics Laboratory of the Wright Aeronautical Laboratory, the STAT (Structural Tailoring of Advanced Turboprops) and the STAEBL (Structural Tailoring of Engine Blades) projects for the propulsion industry funded by NASA Lewis. The three projects use two categories of "optimization engines": nonlinear mathematical programming and optimality criteria.

Recent work on large-scale optimization has focused on:

1) Extending the approximation concepts of Ref. 198 to structural components in the presence of geometric nonlinearities,

2) Integration of the analysis and optimization algorithm in obtaining the design,[82]

3) Shape optimization,[37,83]

4) Full integration of the structure and other disciplines in the optimization of engineering systems (e.g., simultaneous optimization of the structures and controls for large space systems),

5) Large-scale sensitivity analysis, and

6) Decomposition algorithms and multilevel optimization.

The development of efficient computational procedures for large-scale sensitivity analysis has recently attracted considerable attention. The calculation of sensitivity derivatives forms the backbone of many optimization procedures and is the major contributor to the cost and time of optimization of large systems. In addition, sensitivity derivatives have several other applications in structural mechanics, including approximate analysis (and reanalysis) techniques, analytical model improvement, and assessment of design trends, as well as the study of sensitivity of these designs to variations in design variables. A review of the state of the art in sensitivity calculations is contained in Refs. 1 and 86.

Multilevel optimization was introduced in the 1970s and has recently received considerable attention. It is accomplished by means of a formalized linear decomposition of the large design problem into a set of smaller, manageable (nested) subproblems coupled by means of the sensitivity data that measure the change of the subsystem design due to a change in the system design.[205,206] Each subsystem is optimized while holding constant its inputs, received from higher-level subsystems, in the decomposition hierarchy. Different analysis and optimization tools can be used for obtaining the optimal design variables of the subsystems, provided the sensitivity of these variables with respect to the inputs that have been held constant in the optimization are calculated. The sensitivity derivatives are transmitted to the higher-order subsystems to be used there for quick estimates of the changes that will be introduced by modifications. The multilevel approach has been applied to the design of a transport aircraft wing in Ref. 185. Although heuristic decomposition makes large-scale optimization problems solvable, it does not ensure that the true optimum will be determined. Therefore, the success of the multilevel optimization approach greatly depends on the designer's physical understanding of the problem.

## Impact of New Computing Systems

The last two decades have witnessed an explosive growth in computer technology, and the changes forecasted for the next decade will prove to be even greater. This is particularly true with the introduction of novel forms of machine architecture (e.g., multiprocessor systems) and the development of smart computers (e.g., the Japanese fifth-generation project). A description of some of the new and emerging computer systems is given in Refs. 69, 104, 143, and 217. The advances in computer hardware and software technology likely to have the strongest impact on structural mechanics are the following:

#### New Computer Architectures

Increased computer performance can be obtained by using either 1) a single or a few high-speed processor(s), or 2) several (hundreds or thousands of) inexpensive processors to perform the computation.[147] These two classes are usually referred to as supersystems and highly parallel systems, respectively. These two classes and some of the other new computing systems are discussed subsequently.

*Supersystems*

The development of supersystems now spans two generations. Examples of the second-generation supersystems are the CRAY X-MP, the CRAY-2, the CDC CYBER 205, and the NASA Numerical Aerodynamic Simulation (NAS) capability. The current status of the NAS facility is described in Ref. 28. Future supersystems (e.g., ETA-10 and CRAY-3) are expected to have sustained CPU speeds in excess of 1-billion floating-point operations per second (1 gigaflop) before the end of the present decade. These supersystems will make possible new levels of sophistication in modeling structures that were not possible before. Examples are provided by reliability based (stochastic) modeling of structures (to account for probabilistic aspects of geometry, boundary conditions, material properties, and loading) and multidisciplinary analysis and design of structures.

*Highly Parallel Systems*

Highly parallel systems have several synchronous or asynchronous processors performing the computations. In the case of synchronous processors, the processors are identical, are used under common control, and are performing the same operation simultaneously on different data. Examples of parallel machines with synchronous processors are the ICL Distributed Array Processor (DAP), with 1024 processors, at Queen Mary College in London; the Goodyear Aerospace Massively Parallel Processor (MPP), with 16384 processors; and the Connection Machine, with up to 64,000 processors, of the Thinking Machine, Inc. In the case of several asynchronous processors, each processor obeys its own instructions, with some mechanisms for interprocessor communication, and local and global control. Examples of these machines are the HyperCube of the California Institute of Technology and the ULTRA Computer of New York University. The next generation of highly parallel systems is likely to have multiple-level parallelism. The Cedar supercomputer project at the University of Illinois is an example of such architecture. It has multiple clusters of parallel processors with a globally shared memory.[104]

Highly parallel systems will substantially expedite the multidisciplinary design process of structures by allowing the designer to carry out various analysis and design tasks in parallel. The tasks can belong to an individual discipline as well as to other disciplines (such as in multidisciplinary optimization problems).

*Other Parallel Systems*

Other parallel systems with a moderate number of low-cost and midrange processors (performance of the order of 2 MFLOPS for the former and 10–100 MFLOPS for the latter) have been developed. Examples of the former are the ENCORE, FLEXIBLE, SEQUENT, and INTEL systems. Examples of the latter are the ALLIANT and ELXSI systems.

*Minisupercomputers*

A number of new minisupercomputers with performance in the range of 10–100 MFLOPS have been developed. These computers have pipelined, multiunit architectures for vectors and scalar operations. Examples of these machines are the Floating Point System M64 series (M64/30, M64/40, M64/50, and M64/60), the STAR Technologies ST-100, and the CONVEX computer.

**Small Systems**

These include both the new powerful microprocessors and the engineering workstations.

*Microprocessors and Chip Technology*

The trend of ever-increasing the number of devices packaged on a chip has resulted in the miniaturization and increase in speed of microprocessors and minicomputers. The new powerful chips can be used as monitoring systems for the detection, recording, and evaluation of stochastic damage, thereby increasing the mean time between inspection for structural components.

Microcomputers are becoming an integral part of laboratory testing for processing data in support of direct certification of structural components.

*Engineering Workstations with User-Friendly Interface*

Engineering workstations using VLSI 32-bit processor chips and having over 16 Mbytes (or more) of addressable memory have been developed. Examples of such systems are the HP-9000 and the Apollo computers. A superstation with 64-bit processor chips and peak performance of 12 MFLOPS has recently been advertised by Floating Point Systems (M64/330). It is anticipated that by the end of the present decade, the workstations will be desktop computers with sustained speeds of 100 MFLOPS, over 50 Mbytes of addressable memory, over megapixel display, and over megabit transmission rate within local area networks (LAN's). Moreover, sophisticated user-friendly software and hardware interfaces will be developed that will improve the productivity of the analysts. Future options are likely to be multiwindow and will be controlled by voice or mouse. These devices can substantially reduce the analysis and design time of practical structures. The structural model can be changed whenever desired via voice or mouse.

*Special-Purpose Computing Hardware*

With the continued reduction in the cost of computer hardware, a number of special-purpose computers have emerged that offer increased speed for specific problems when compared with general-purpose computers. Examples of these special-purpose computers are the finite-element machine (FEM) developed at NASA Langley Research Center, and the Navier-Stokes Computer (NSC) being developed at Princeton University. The FEM was developed as an experimental research tool for demonstrating the potential of highly parallel architectures for high-speed yet low-cost solutions of finite-element problems.[211] The NSC is a parallel processing machine with a hypercube architecture for simulating the two-dimensional nonsteady viscous flows. A description of the NSC machine is given in Ref. 161.

There is a growing trend of moving from software-based processing to hardware-based processing, and it is likely that the classical linear finite-element analysis will become a hardware function in the future.

**AI Knowledge-Based and Expert Systems**

AI-based expert systems, incorporating the experience and expertise of practitioners, have high potential for the modeling, analysis, and design of structures. These systems can aid the structural analyst in the initial selection and adaptive refinement of the model, as well as in the selection of the appropriate algorithm used in the analysis. Expert systems can also aid the structural designer by freeing him from such routine tasks as the development of process and material specifications. A review of the capabilities of some of the currently available expert systems and their limitations is given in Refs. 66, 73, and 74. A description of a knowledge-based system used as a modeling aid for aircraft structures is given in Ref. 214.

**Large Powerful Data Management Systems and Databases**

Future engineering software systems are likely to have the basic analysis software (such as data management, control, etc.) as part of the software infrastructure and the discipline specifics (such as the finite-element properties of the structure) as part of application software. Advanced database concepts, such as the relational database management system, RIM,[63,193] will provide improved multidisciplinary coordination and will facilitate integrating a structural analysis program into CAD/CAM systems.

## Future Directions for Research

Computational structural mechanics is likely to play a significant role in the future development of the structures technology, as well as in the multidisciplinary design of large systems, and for this to happen major advances are needed in a number of key areas of CSM. To this end, there are a number of primary and secondary pacing items that must be addressed by the research community. Also, advantage should be taken of the recent and future developments in other fields of computational mechanics (notably fluid dynamics), and of the emerging field of computational mathematics.[190]

The primary pacing items include: 1) prediction and analysis of failure of structural components made of new materials, 2) development of effective computational strategies and solution methodologies for large-scale structural calculations, and 3) assessment of the reliability of numerical response predictions and their adaptive improvement. The secondary pacing items include: 1) modeling of complicated structures, 2) predata and postdata processing, and 3) integration of analysis programs into CAD/CAM systems. These pacing items are discussed subsequently.

**Primary Pacing Items**

*Prediction and Analysis of Failure of Structural Components Made of New Materials*

Future structural systems require the development of high-performance, new materials such as high-temperature materials (e.g., superalloys, metal-matrix composites, and carbon/carbon composites) for hypersonic vehicles; piezoelectric composites; electronic and optical materials for space applications. Consequently, practical numerical techniques are needed for predicting the failure initiation and propagation in structural components made of these materials, in terms of measurable and controllable parameters. This, in turn, requires the availability of accurate microstructurally based constitutive descriptions, failure criteria, and damage theories. Also, more realistic characterization of interface phenomena, such as contact and friction, is needed.

*Development of Computational Strategies and Solution Methodologies for Large-Scale Structural Calculations*

Examples of large-scale problems for which solutions are not economically feasible on present-day large computers are dynamics of large flexible structures incorporating the effects of joint nonlinearities and hysteritic damping; interaction problems of large structures with harsh environments, such as soils under earthquake excitation; thermal fields or electromagnetic fields; and large-scale multidisciplinary design problems. The solution of these problems requires:

1) Development of special methodologies and strategies for large-scale problems, such as the time-honored divide and conquer strategy. More specifically, for a wide class of large-scale analysis problems, the hybrid techniques based on splitting and nesting (or reduction) appear to have high potential. These techniques can be thought of as general computational strategies for: a) generating the response of a complex structural system using large perturbations from that of a simpler system, and b) obtaining sensitivity information (derivatives of the various response quantities of the structure with respect to preselected set of control parameters).

2) Exploiting the capabilities of new and emerging computing systems (e.g., parallel- and vector-processing capabilities of multiprocessor computers).

Much work has recently been devoted to the development of vectorized and parallel numerical algorithms for performing the matrix operations and solution of algebraic equations (see, for example, Refs. 167 and 228). Also, compilers have been developed for the automatic vectorization of scalar codes (e.g., CFT and CIVIC compilers on the CRAY computer) and preprocessors are now available for the automatic multitasking of user-specified portions of scalar codes (e.g., PREMULT on the CRAY). However, experience gained with these software tools has shown that direct conversion of scalar codes does not result in realizing the full potential of the powerful supercomputers. The effective use of the new supercomputers requires maximizing the degree of parallelism at each level in the analysis and design process. As an example, for nonlinear dynamic problems, this includes exploiting the parallelism at the following levels: a) formulation level (through the use of primitive-variable formulation or mixed finite-element models); b) analysis level (through spatial substructuring and temporal partitioning with minimization of interfaces); c) numerical algorithm level (through the use of operator splitting techniques); and d) implementation level (through the use of multitasking on the CRAY X-MP and CRAY 2 computers). Also, efficient vectorization should be attempted for each of the subtasks. An assessment should be made of the effect of the different partitioning strategies on the overhead expended in scheduling, communication, and synchronization of the different tasks. An optimal strategy is one which, among other things, minimizes this overhead.

*Assessment of Reliability and Adaptive Improvement of Response Predictions*

In spite of the considerable attention devoted by engineers and mathematicians to the subject of error estimation and control, none of the large-scale commerical finite-element systems has facilities for error estimation or adaptive improvement. To remedy this situation, major advances are needed in the theory, strategies, and algorithms for implementation of error estimation and control. These advances include:

1) Development of reliable measures for estimating the errors in the predictions of the computational model of a structure. These errors are due to the simplifying assumptions made in abstracting the mathematical model from the real structure, uncertainties in the input information of the mathematical model, and numerical discretization of the continuous mathematical model of the structure.

2) Development of efficient adaptive improvement strategies as well as efficient computer implementation of these strategies (through the use of novel computer science concepts for data management).

3) Systematic assessment of the postprocessing techniques used for improving the accuracy of derivative calculation (stresses and strains in displacement finite-element models) is needed. Also, the possibility of using these techniques for error sensing should be investigated.

4) Selection of meaningful benchmark problems, which have the essential features of practical problems to test the theory and the effectiveness of adaptive strategies developed.

The maturation of the technology of estimation and control of discretization errors, and the incorporation of this technology into general-purpose finite-element systems will allow the analyst to select only: a) the initial discrete model, which is sufficient to resolve the topology of the structure; and b) the error measure and the tolerance. Then the finite-element system can automatically refine the model until the selected error measure falls below the prescribed tolerance. The strategy for adaptive improvement can either be specified by the user or automatically selected by the program (possibly with the aid of an AI-based expert system) in such a manner as to minimize the cost of the analysis.

**Secondary Pacing Items**

Of importance to the users of CSM are three secondary pacing items, which are receiving increasing attention within the research community. These are: 1) modeling of complex structures; 2) predata and postdata processing, and 3) integration of analysis programs into CAD/CAM systems.

*Modeling of Complex Structures*

One of the most important steps for the accurate prediction of the response of a structure is the proper selection of the mathematical and discrete models. Hence, there is a need for the development of automatic model generation facilities as well as knowledge-based and expert systems that can help the analyst/designer in the initial selection of the model, its adaptive refinement, and the interpretation of results. An example of the recent work on automatic model generation is given in Ref. 201.

*Predata and Postdata Processing*

Because of the enormous amount of data that is required for defining a complex structure and characterizing its response, there is a need for developing optimal predata and postdata processing procedures. Processing of bulk data can currently be done effectively using high-resolution, high-throughput graphics devices. Also, generation of three-dimensional color movies (film and video) has been invaluable for visualizing the dynamic response of structures. However, effective visualization and interpretation of results from large multidimensional data sets require the development of postprocessing facilities, including tools for data interpretation (representation of complex multidimensional data sets, enhancing human/computer interaction (e.g., through speech commands as well as tactile and visual interfaces), and data understanding (through the use of novel graphic and animation techniques).

*Integration of Analysis Programs into CAD/CAM Systems*

Much effort is now being directed to the integration of analysis programs into CAD/CAM systems.[31] With the trend of moving from software-based processing to hardware-based processing, some of the analysis modules are likely to become hardware functions. An investigation of the interface between, and optimal combination of, software and hardware functions is needed.

## Concluding Remarks

The status and some recent developments of computational structural mechanics are summarized. Discussion focuses on a number of aspects, including computational needs for future structures technology and their implications on computational structural mechanics (CSM), advances in computational models for material behavior, discrete element technology, quality assessment and control of numerical simulations of structural response, hybrid analysis techniques, and techniques for large-scale optimization, and the impact of new computing systems on CSM.

Some of the primary and secondary items that pace the progress of CSM are identified and are, therefore, recommended as future directions for research. The primary pacing items include prediction and analysis of failure of structural components made of new materials, constitutive modeling of new materials, development of computational strategies and solution methodologies for large-scale structural calculations, and assessment of reliability of response predictions and their adaptive improvement. The secondary pacing items include modeling of complex structures, predata and postdata processing, and integration of analysis software into CAD/CAM systems.

Computational structural mechanics used in a heuristic mode has greatly enhanced our understanding of physical phenomena. An example of this is the study of the implications of various microstructural mechanisms of inelastic deformation on the macroscopic response. CSM can also enhance the understanding of the mathematics of complex response phenomena and allows us to penetrate into unexplored regions of the mathematical theory of structures. An example of this is nonlinear chaotic dynamics of structural systems whose history is sensitive to initial conditions. Despite the tremendous success of CSM in explaining complex response characteristics of structures, it is the authors' belief that CSM will not replace either experimental or analytical structural mechanics as a research tool, but it will complement and supplement them invaluably.

## Acknowledgments

The work of A. K. Noor was supported by NASA Langley Research Center, and the work of S. N. Atluri was supported by both the Air Force Office of Scientific Research and the Office of Naval Research. The authors wish to acknowledge useful discussions and suggestions by many of their colleagues, including H. Armen, J. K. Bathe, T. Belytschko, L. Berke, C. C. Chamis, T. A. Cruse, C. A. Felippa, R. T. Haftka, T. J. R. Hughes, W. K. Liu, J. T. Oden, J. Padovan, K. C. Park, L. A. Schmit, J. Sobieski, J. Starnes, O. O. Storaasli, V. B. Venkayya, and K. J. Willam. The constructive comments of the reviewers are also appreciated.

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